Coupling factors of a 3-winding transformer

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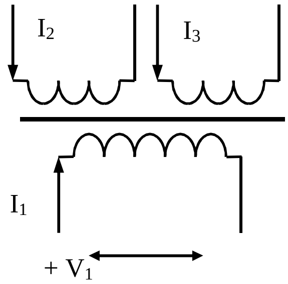

When two of the coupling factors of the 3-winding transformer shown here are known, the third is confined to a certain range. How to find that range?
For winding 1 of the transformer we have the equation:

$$V_1 = j\omega.(L_1 I_1 + M_{12} I_2 + M_{13} I_3)$$

where V_1 is the voltage across winding 1,
L_1 is the open circuit inductance of winding 1,
ω is the frequency of a sinusoidal current,
I_j are the winding currents and
M_{ij} are the mutual inductances between the windings: $M_{i,j} = k_{i,j}\sqrt{L_i L_j}$.

Scaling the currents I_j to the open circuit inductances of their windings and to the frequency as: $i_j = j\omega I_j\sqrt{L_j}$

and scaling the voltages V_i likewise as: $v_i = \frac{V_i}{\sqrt{L_i}}$,

the behaviour of the transformer can now be described by means of a coupling matrix K, where $\underline{v} = K \bullet \underline{i}$, as follows:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & k_{12} & k_{13} \\ k_{12} & 1 & k_{23} \\ k_{13} & k_{23} & 1 \end{bmatrix} \bullet \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

To find the currents in all windings we have to solve the equation: $\underline{i} = K^{-1} \bullet \underline{v}$.
Example: for finding short-circuit currents in windings 2 and 3 with a voltage present at winding 1 we can write: v_1=1, v_2=0, v_3=0 and calculate $\underline{i}$.
This requires matrix inversion and a non-zero determinant D of matrix K:

$$D = \left[1 - k_{12}^2 - k_{13}^2 - k_{23}^2 + 2k_{12}k_{13}k_{23}\right]$$

Physically valid situations must have $D > 0$.
(Example: when all values of k are 0.9 we have $D = 0.028$).

The determinant gives a criterion for the third coupling factor when the first two are given. Let k_{12} and k_{13} be given. A valid range for k_{23} can then be found by solving the second order equation for $D = 0$.
The two solutions are:

$$k_{23,\min,\max} = k_{12}k_{13} \pm \sqrt{k_{12}^2 k_{13}^2 - k_{12}^2 - k_{13}^2 + 1}$$

A special case is $k_{12} = k_{13} = k_1$ and $k_{23} = k_2$. This leads to:

$k_{2,min} = 2(k_1)^2 - 1$ and obviously: $k_{2,max} = 1$

Example: a transformer has a primary winding 1 and two equal secondary windings 2 and 3. Let $k_1 = 0.9$. The coupling factor between the secondary windings must satisfy: $1 > k_2 > 0.62$.
Note that a zero coupling factor k_2 is only possible if $k_1 < \frac{1}{2}\sqrt{2}$.

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